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Minimum Weight Web-Core Sandwich Panels Subjected to Uniaxial Compression

Jack R. Vinson*
University of Delaware, Newark, Del.

AND

Sidney Shore†
University of Pennsylvania, Philadelphia, Pa.

Analytical methods are developed by which web-core sandwich panels can be designed with minimum weight while retaining their structural integrity (i.e., structurally optimum) for a given load index, panel length and width, and specified face and web materials. These methods also provide a means for rational material selection through the comparison of weights of the minimum weight construction for various material system as a function of load index, and an assessment of weight penalties associated with the use of commercially available thicknesses. The methods account for both isotropic and orthotropic face and core material as well as various boundary conditions. Numerical results clearly show the weight savings associated with the use of second generation composite materials such as boron-epoxy and graphite-epoxy.

Nomenclature

a = length of sandwich panel in the x direction, in.
$\bar{A}_c$ = core area per unit width, in.
b = width of sandwich panel in the y direction, in.
d_f = distance between web elements (see Fig. 1), in.
D_{QY} = transverse shear stiffness, per unit width in the x direction, lb.-in.
E_{ij} = modulus of elasticity
E_{0i} = orthotropic modulus defined by Eq. 12, psi
$\bar{E}$ = reduced modulus (ηE), psi
h_c = core depth, in.
$\bar{I}_c$ = area moment of inertia of the core about the panel centroidal axis per unit width, in.^{3}
$\bar{I}_f$ = area moment of inertia of the faces considered as membranes about the centroidal axis of the panel, per unit width, in.^{3}
K = buckling coefficient
N_x = compressive in-plane load per unit width, lb/in.
R = quantity defined generally as $1 + 2\ (\rho_c/\rho_f)(E_{fx}/E_{cx})$
t_c = thickness of web element, in.
V = transverse shear flexibility parameter in the y direction, defined in Eq. 4.
W = weight of the over-all sandwich construction per unit of planform area, psi
W_{ad} = weight of the material joining the faces to the core per unit of planform area, psi
W_i = weight of the core or face per unit of planform area, psi
ν_{jki} = Poisson's ratio
η = plasticity reduction factor
ρ_i = specific weight, lb/in.^{3}
σ_i = critical stress
θ = angle the web element makes with a line normal to the faces, see Fig. 1

Subscripts

i = c,f
j = x,y
k = $x,y\ (\neq j)$

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* Professor and Chairman of Mechanical and Aerospace Engineering. Associate Fellow AIAA.

† Professor and Chairman of the Graduate Group Committee in Civil and Urban Engineering.

Background

STRUCTURAL optimization to obtain minimum weight configurations has been the center of considerable study for decades. Texts such as the notable work of Shanley,[1] the numerous publications of Schmit and others attest to the continued interest in this effort. An inclusive bibliography on the subject of structural methods and optimization of sandwich panels up to 1965 is given in Ref. 2.

To discuss all or even a significant fraction of all optimizations is beyond the scope of this paper. However, for completeness the previous work of the present authors is recorded concerning optimization of honeycomb and solid core sandwich panels subjected to a variety of loads including in-plane compression, in-plane shear loads, and lateral loads (Refs. 3 and 4), and the optimization of corrugated core sandwich panels subjected to uniaxial compression, in-plane shear loads, and lateral loads (Refs. 5, 6, 7 and 8). Portions of Refs. 2 and 3 were presented at the Fifth U.S. National Congress of Applied Mechanics in 1966.

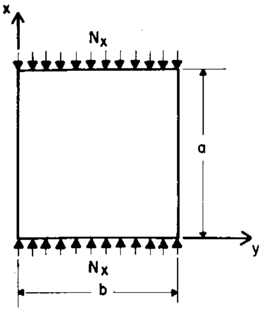

Fig. 1 Cross section of web core construction.

Introduction

Consider a flat web-core sandwich panel, generalized to include some arbitrary angle θ, as shown in Fig. 1. The over-all geometry and loading is given in Fig. 2. There are five geometric variables with which to optimize; namely, the core depth h_c, the web thickness t_c, the face thickness t_f, the angle the web makes with a line normal to the faces (θ), and the distance between web elements d_f.

The panel is considered to fail if any of the following instabilities occur: overall panel instability, local face buckling in the region from A to B, local face buckling in the region B to C, and web element buckling (see Fig. 1). Overstressing is not a failure mode because for critical stresses above the proportional limit of the face and core material, a reduced modulus incorporating a plasticity reduction factor is used.

Hence, there are five geometric variables and four modes of failure. To describe the instabilities mathematically the analytical expression used in each case is the best available in the literature.

Elastic and Geometric Constants

The elastic and geometric constants for the web-core construction can be determined from those given in more general form by Libove and Hubka.[9] The core area per unit width and the moment of inertia of the core about the centroidal axis per unit width are given by:

$$\bar{A}_c = t_c h_c/(d_f + h_c \tan\theta) \cos\theta \tag{1}$$

$$\bar{I}_c = t_c h_c^3/12 \cos\theta(d_f + h_c \tan\theta) = \bar{A}_c h_c^2/12 \tag{2}$$

The transverse shear stiffness of the core, per unit width in the x direction (D_{QY}) of an element of the sandwich panel cut by two $y - z$ planes is seen to be negligible, due to the lack of structural continuity of the web core; hence, following Libove and Hubka,[9] as well as Seide[10]

$$D_{QY} \to 0 \tag{3}$$

Hence, the transverse shear flexibility parameter in the y direction is given by:

$$V = \pi^2 E_f \bar{I}_f / b^2 D_{QY} \to \infty \tag{4}$$

This is not to say that the construction as a whole has no transverse shear stiffness, but rather that the stiffness that does exist depends upon the faces to provide the continuity (see Fig. 1). In fact Seide discusses this case and states that for the case of $D_{QX} = \infty$ and $D_{QY} = 0$, the compressive buckling load is finite, rather than being equal to zero, and varies with plate aspect ratio.

It should be noted that when $\theta \neq 0$ and $d_f = 0$, the construction has a continuous core, called corrugated core or truss core, with the result that $D_{QY} \neq 0$, and there is a specific value of θ for which the weight is a minimum. This construction is treated in Ref. 6.

The area moment of inertia per unit width of the faces considered as membranes, with respect to the sandwich middle surface, per unit width, is seen to be

$$I_f = t_f h_c^2/2 \tag{5}$$

Since $t_f \ll h_c$, the core depth h_c can be taken as the distance between the centerlines of the individual faces.

Governing Equations

Over-all Panel Instability

The best expression applicable to the over-all instability of a web-core sandwich panel composed of isotropic materials under uniaxial compressive loads is derived by Seide,[10] and given as follows:

$$N_x = \pi^2 E_f \bar{I}_f K / 2b^2 \tag{6}$$

where K is the buckling coefficient derived and plotted in Figs. 2 and 4 of Ref. 10 for various boundary conditions. For simply supported unloaded edges, and this construction can be given explicitly by

$$K = \frac{[(1/\beta) + \beta^2]}{1 - \nu_f^2 + 2(1 + \nu_f)\beta^2} + \frac{E_c \bar{I}_c}{E_f \bar{I}_f \beta^2} \tag{7}$$

where $\beta = a/b$.

There is no published analytical expression describing the over-all panel instability of web-core sandwich panels utilizing orthotropic materials subjected to uniaxial compressive loads. However, it is not difficult to deduce the form of the equation by observing the differences in the analogous expressions for honeycomb core sandwich panels for isotropic and orthotropic materials.

From the expressions in Ref. 11, it is seen that when the over-all instability expressions are written for orthotropic materials E_f is replaced by $(E_{fx}E_{fy})^{1/2}$ when flexural properties are considered, while E_f is replaced by E_{fx} when extensional properties are involved. It is therefore deduced that for web-core panels utilizing orthotropic materials Eq. (6) becomes, utilizing Eq. (5)

$$N_x = \pi^2 (E_{fx}E_{fy})^{1/2} t_f h_c^2 K / 2b^2 \tag{8}$$

It is also hypothesized that Figs. 2 and 4 of Ref. 10 may be extended to find the buckling coefficient K for the orthotropic construction if $E_c\bar{I}_c/E_f\bar{I}_f$ is determined by

$$\frac{E_c\bar{I}_c}{E_f\bar{I}_f} = \frac{(E_{cx}E_{cy})^{1/2} t_c h_c}{6(E_{fx}E_{fy})^{1/2} t_f (d_f + h_c \tan\theta) \cos\theta} \tag{9}$$

For the optimum construction Eq. (9) is a function of material properties only.

Likewise Eq. (7) may be extended to orthotropic construction by replacing $(1 - \nu_f^2)$ by $(1 - \nu_{xyf}\nu_{yxf})$, and $(1 + \nu_f)$ by $(1 + \nu_{xyf})$.

Face Plate Instability

From Fig. 1 it is seen that the face elements A to B and C to D may each undergo an elastic instability under uniaxial compressive loads. Since the unloaded edge supports are not known precisely, it is conservative to assume a simple support. Since for almost all panels, the panel length a is much greater than the width, which is the distance A to B or C to D, the buckling coefficient is taken as 4.

For the faces made of an orthotropic material the expression for the critical face stress given by Timoshenko and Gere[12] is used. In terms of the quantities defined in Fig. 1, the critical stress can be written as follows for the region A to

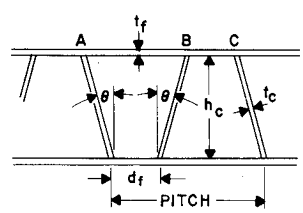

Fig. 2 Planform of panel.

B:

$$\sigma_{f1} = \frac{\pi^2 E_{0f} t_f^2}{3(1 - \nu_{xyf}\nu_{yxf})(d_f + 2h_c \tan\theta)^2} \tag{10}$$

and in the region B to C:

$$\sigma_{f2} = \pi^2 E_{0f} t_f^2 / 3(1 - \nu_{xyf}\nu_{yxf}) d_f^2 \tag{11}$$

where

$$2E_{0i} = (E_{ix}E_{iy})^{1/2} + \nu_{yxi}E_{ix} + 2G_{xyi}(1 - \nu_{xyi}\nu_{yxi}). \quad (i = c,f) \tag{12}$$

Web Plate Instability

Similar to the above, the plate instability equation for a web element composed of an orthotropic material can be written as

$$\sigma_c = \pi^2 E_{0c} t_c^2 \cos^2\theta / 3(1 - \nu_{xyc}\nu_{yxc}) h_c^2 \tag{13}$$

Load Stress Relationship

For the construction of Fig. 1, it is seen that the load/in., N_x, is related to the face and web stresses, σ_f and σ_c, by the following relationship

$$N_x = \sigma_c \bar{A}_c + 2\sigma_f t_f$$

By equating the axial strains in the core and face to insure compatibility in the over-all construction, the following relationship is easily derived:

$$\sigma_c = \sigma_f E_{cx}/E_{fx} \tag{14}$$

Thus from Eqs. (12), (13) and (1), the load/in. N_x is related to the face stress as follows

$$N_x = \sigma_f \{[E_{cx} t_c h_c / E_{fx}(d_f + h_c \tan\theta)] + 2t_f\} \tag{15}$$

Both Eqs. (14) and (15) hold only when stresses in the face and core are below the proportional limit of each material, where Hooke's Law applies. Above the proportional limit of either material an iterative procedure would be needed to insure compatibility in determining an analogous relationship to Eq. (14), employing some reduced moduli $\bar{E}_{ci}$ and $\bar{E}_{fi}$ involving a plasticity reduction factor.

If both core and face materials are the same $\sigma_c = \sigma_f$, then the procedures which follow apply for stresses above the proportional limit if a suitable plasticity reduction factor is used with the modulus of elasticity.

Weight Relation

From Fig. 1, it is seen that

$$W = 2\rho_f t_f + \rho_c \bar{A}_c + W_{ad} \tag{16}$$

where ρ_c and ρ_f are the specific weight of the core and face material, respectively; and W and W_{ad} are the panel weight per square inch of planform area and the adhesive or other material used to join face to core, respectively.

Structural Optimization—General

The philosophy of structural optimization is as follows: a truly optimum structure is one which has a unique value for each geometric variable, within the class of structures being studied, for each material system, which will result in the minimum possible weight for a specified loading condition, and yet maintain its structural integrity. In this case the optimum structure will have the characteristic that the panel will become unstable in all four buckling modes simultaneously, a criterion confirmed by Schmit[13] and others for simple loadings such as the one studied here.

The equations with which to optimize are: the four buckling Eqs., (8), (10), (11), and (13); the load-stress relationship, Eq. (15); and the weight relationship, Eq. (16). The known quantities for any optimization are N_x, a, b, and the material properties of both core and face material. The buckling coefficient K is a constant depending on the value of Eq. (9), and is given by Eq. (7). The unknown variables to be determined are t_f, h_c, t_c, d_f, θ, σ_f, and $(W - W_{ad})$. With six equations and seven unknown variables, a seventh equation is obtained by placing the weight equations in terms of one variable only, and setting the derivative of weight with respect to that variable to zero. Thus we find the value of that variable, and subsequently the value for each other variable, which results in minimum weight.

At the outset, independent of the material system, it is clear that in executing the above optimization philosophy, that from Eq. (9) and Eq. (10) for optimum construction of the web-core sandwich

$$\theta = 0° \tag{17}$$

This is intuitively obvious. The result is that all other expressions used for optimization are simplified, and the construction show in Fig. 1, with $\theta = 0°$ now assumes the familiar web-core configuration.

Structural Optimization of Panels with Faces and Core of Different Orthotropic Materials

Proceeding with the optimization procedure described above the following expressions are obtained for the optimum (minimum weight construction). First, one obtains the "universal relationship" relating applied load index (N_x/b) to a unique value of face stress σ_f, as a function of given material properties, for the optimum construction

$$\frac{N_x}{b} = \frac{(6)^{1/2}(1 - \nu_{xyf}\nu_{yxf})^{1/4}(1 - \nu_{xyc}\nu_{yxc})^{1/4}}{(E_{0f}E_{0c})^{1/4}(E_{fx}E_{fy})^{1/4}\pi^2 K^{1/2}} \times \left(\frac{E_{cx}}{E_{fx}}\right)^{7/4}\left(\frac{\rho_f}{\rho_c}\right) R^{3/2}\sigma_f^2 \tag{18}$$

where $R = 1 + 2(\rho_c/\rho_f)(E_{fx}/E_{cx})$ and K is determined from Figs. 2 or 4 of Ref. 10 in which $V \to \infty$ and for the optimum construction

$$\frac{E_c\bar{I}_c}{E_f\bar{I}_f} = \frac{1}{6}\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_{cx}E_{cy}}{E_{fx}E_{fy}}\right)^{1/2}$$

In a panel with all edges simply supported K is given as follows, using the above expression for $E_c\bar{I}_c/E_f\bar{I}_f$

$$K = \frac{[(1/\beta) + \beta^2]}{1 - \nu_{xyf}\nu_{yxf} + 2(1 + \nu_{xyf})\beta^2} + \frac{E_c\bar{I}_c}{E_f\bar{I}_f\beta^2} \tag{19}$$

where $\beta = a/b$.

The significance of the universal relationship given by Eq. (18) is that for a panel of a given length a, with b, compressive load per unit width N_x, and material system, if the panel is designed for a face stress of either higher or lower than that given by Eq. (18), the panel will be heavier than one employing Eq. (18) with the following geometric properties. Making use of Eq. (18), the geometry of the panel for minimum weight construction can be written as:

$$\frac{h_c}{b} = \left\{2\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_{cx}}{E_{fx}}\right)\frac{R\sigma_f}{(E_{fx}E_{fy})^{1/2}\pi^2 K}\right\}^{1/2} \tag{20}$$

$$\frac{h_c}{d_f} = \left\{\left(\frac{\rho_f}{\rho_c}\right)^2\left(\frac{1 - \nu_{xyf}\nu_{yxf}}{1 - \nu_{xyc}\nu_{yxc}}\right)\left(\frac{E_{0c}}{E_{0f}}\right)\left(\frac{E_{fx}}{E_{cx}}\right)\right\}^{1/4} \tag{21}$$

$$\frac{t_c}{b} = \frac{6^{1/2}}{\pi^2 K^{1/2}}\frac{(1 - \nu_{xyc}\nu_{yxc})^{1/2}}{E_{0c}^{1/2}(E_{fx}E_{fy})^{1/4}}\left(\frac{\rho_f}{\rho_c}\right)^{1/2}\left(\frac{E_{cx}}{E_{fx}}\right) R^{1/2}\sigma_f \tag{22}$$

$$h_c/d_f = (\rho_f/\rho_c)(t_f/t_c) \tag{23}$$

$$(W - W_{ad})/b = 3\rho_f(t_f/b) \tag{24}$$

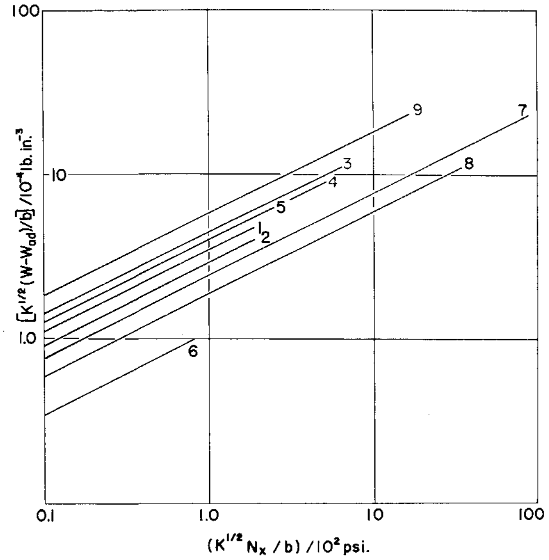

Fig. 3 Weight parameter as a function of load index.

Other alternative expressions for the optimum construction are given in Ref. 5.

It can also be shown easily that even in this general material system, the ratio of the face weight W_f to the core weight W_c for the optimum construction is:

$$W_f/W_c = 2 \tag{25}$$

Note that this is independent of applied load, material system and panel boundary conditions.

Structural Optimization of Panels with Faces and Core of Different Isotopic Materials

The resulting expression for the optimum construction for this material system can be obtained from Eqs. (18–25) by allowing $E_{ix} = E_{iy} = E_{0i} = E_i$ and $\nu_{xyi} = \nu_{yxi} = \nu_i$ where $i = c,f$.

Structural Optimization of Panels with Faces and Core of the Same Orthotropic Construction

The resulting expressions for the optimum constructions for this material can be obtained from (17–23) by letting each quantity $(\ )_f = (\ )_c = (\ )$. The results are

$$\frac{N_x}{b} = \frac{9(2)^{1/2}(1 - \nu_{xy}\nu_{yx})^{1/2}\sigma^2}{\pi^2 K^{1/2} E_0^{1/2}(E_x E_y)^{1/4}} \tag{26}$$

where K is determined from Fig. 2 or 4 of Ref. 2 or, for a simply supported panel, from Eq. (19), in which

$$E_c\bar{I}_c/E_f\bar{I}_f = \tfrac{1}{6}$$

Making use of Eq. (26) to obtain σ, the other geometric variables are obtained easily from the following:

$$h_c/b = \{6\sigma/\pi^2 K(E_x E_y)^{1/2}\}^{1/2} \tag{27}$$

$$d_f = h_c \tag{28}$$

$$\frac{t_c}{b} = \frac{3(2)^{1/2}(1 - \nu_{xy}\nu_{yx})^{1/2}\sigma}{\pi^2 K^{1/2}(E_x E_y)^{1/2} E_0^{1/2}} \tag{29}$$

$$t_f/t_c = 1 \tag{30}$$

$$\frac{W - W_{ad}}{b} = \frac{\rho}{\sigma}\left(\frac{N_x}{b}\right) = 3\rho\frac{t_f}{b} = 3\rho\frac{t_c}{b} \tag{31}$$

It is seen that for a panel in which the same orthotropic material is used in both face and core, the optimum geometry face and core of the same thickness ($t_c = t_f$), and square cells ($d_f = h_c$).

Structural Optimization of Panels with Faces and Core of the Same Isotopic Material

The expressions for optimum construction can be easily determined from Eqs. (26–31) by letting $(\ )_x = (\ )_y = (\ )$, and are recorded below for ease of use.

With this material system, since the material is isotropic and the face and core are equally stressed, the expressions can be employed for loads resulting in stresses above the proportional limit by utilizing a suitably defined plasticity reduction factor η, such that $\bar{E} = \eta E$. Thus, in the following $\bar{E}$ is used to denote that the expressions are valid in the range of inelastic deformations.

$$N_x/b = 9(2)^{1/2}(1 - \nu^2)^{1/2}\sigma^2/\pi^2\bar{E}K^{1/2} \tag{32}$$

$$h_c/b = d_f/b = (6\sigma/\pi^2 K\bar{E})^{1/2} \tag{33}$$

$$t_f/b = t_c/b = 3(2)^{1/2}(1 - \nu^2)^{1/2}\sigma/\pi^2 K^{1/2}\bar{E} \tag{34}$$

$$\frac{W - W_{ad}}{b} = \frac{3(2)^{1/4}\rho(1 - \nu^2)^{1/4}}{\pi K^{1/4}\bar{E}^{1/2}}\left(\frac{N_x}{b}\right)^{1/2} = \frac{\rho}{\sigma}\left(\frac{N_x}{b}\right) = 3\rho\frac{t_f}{b} = 3\rho\frac{t_c}{b} \tag{35}$$

Again it is seen that web and face have the same thickness and that the cells are square for the optimum construction. Note also that the isotropic material which has the highest ratio of $E^{1/2}/\rho$ is the material which will result in the least weight panel in the elastic range.

Example Calculation

As an example of the use of the optimization procedures derived above, curves of the weight function $K^{1/2}(W - W_{ad})$ as a function of load index $(K^{1/2}N_x/b)$ are plotted in Fig. 3 for the following materials:
1) 7075-T6 Aluminum (clad) [Ref. 14, Fig. 3.2.7.1.6(a), p. 116]; 2) S994-181 HTS glass fabric, ERSB-0111 Resin [Ref. 15]; 3) 143 glass fabric laminate with polyester resin (MIL-R-7575) [Ref. 16, Fig. 2–59, pp. 2–53]; 4) 143 glass fabric laminate with epoxy resin (MIL-R-9300) [Ref. 16, Figs. 2–64, pp. 2–54]; 5) 181 glass fabric laminate with epoxy resin (MIL-R-9300), [Ref. 16, Figs. 2–63, pp. 2–54]; 6) cross rolled beryllium[17]; 7) unidirectional boron fibers with epoxy resin; 8) unidirectional Thornel-40 graphite fibers with epoxy resin-, and 9) AISI 4340 steel, 200,000 psi yield strength [Ref. 14, Fig. 2.2.2.1.4(c), p. 32].

In each case the load index is taken only to an optimum stress equal to the yield strength of the material system. The factor $K^{1/2}$, appearing as it does in both the ordinate and abscissa, permits the weight comparison for various material systems regardless of boundary conditions and aspect ratio a/b.

It is clearly seen that the beryllium results in the lowest weight construction, but the yield strength is reached at a low value of load index. It is also clearly seen that not only do boron-epoxy and graphite-epoxy materials provide lower weight construction than glass reinforced plastic systems, and steel and aluminum, but of perhaps equal significance, higher loads can be carried by graphite epoxy and boron-epoxy panels of given size and edge support conditions than any other material system.

Conclusions

One benefit derived by the development of methods of analysis for optimum structures, other than reasons given above, is that it enables the designer to compare the absolute minimum weight construction with the construction utilizing commercially available thicknesses that approximate the actual optimum dimensions. In this way he can more rationally assess the following: the weight penalty of using commercially available sizes vs the cost penalty of using noncommercially available sizes to obtain minimum weight. Obviously, this is a function of the specific application.

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Jet Noise Excitation of an Integrally Stiffened Panel

MERVYN D. OLSON*
University of British Columbia, Vancouver, Canada
AND
GARRY M. LINDBERG†
National Aeronautical Establishment, National Research Council, Ottawa, Canada

The free vibrations and random response to jet noise of an integrally stiffened five bay panel have been studied both theoretically and experimentally. A finite element approach was used to represent the panel for both parts of the study, and the predictions were verified by measurements on a model panel integrally machined from solid Aluminum stock. The comparison between predicted and measured vibration modes and frequencies revealed good correlation of frequencies while the correlation of mode shapes was only fair, especially for higher modes. The predicted modes and frequencies were used in a modal analysis of the panel's response to jet noise with a consistent finite element method being introduced to calculate the required cross spectral modal force terms. Quantitative agreement between predicted and measured rms stresses and displacements was realized, whereas only qualitative agreement was obtained for the associated spectra.

Nomenclature

a_l,b_l,c_l = finite-element dimensions, Fig. 9
c_o = speed of sound
$e_i,\{E\}$ = polynomial coefficients, Eq. (9)
$f_j(x,y)$ = jth mode shape
$\{F(\xi,\eta)\}$ = column vector of polynomial terms, Eq. (7)
$H_j(\omega)$ = complex admittance for jth mode
$I_{jk}(\omega)$ = modal force cross spectral matrix, Eq. (4)
m_j = generalized mass for jth mode
p_o = root mean square pressure
$\{P_0\},\{P_1\},\{P_2\}$ = consistent finite-element load vectors
$[Q(\omega)]$ = cross spectral matrix for generalized coordinates, Eq. (17)
$R(\bar{x},\bar{y},\tau)$ = noise correlation function, Eq. (2)
w = panel and finite-element displacement
$w_{\rm rms}$ = root mean square displacement
$\{W\}$ = column vector of generalized displacements for finite element, Eq. (8)
ζ_j = damping ratio for jth mode

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* Assistant Professor, Dept. of Civil Engineering. Member AIAA.

† Associate Research Officer, Structures and Materials Laboratory. Member AIAA.